Normal-Mode Analysis of Ferrite-Coupled Lines Using Microstrips or Slotlines

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***Abstract*— A normal-mode analysis of a pair of axially-magnetized ferrite-coupled lines (FCL) is presented for the first time. This normal-mode method, as opposed to the coupled-mode method, will permit optimization of propagation characteristics and impedance-matching. Also, the finite element solution of the normal modes can be used to obtain the field distribution to assist suitable placement of ferrite for device applications. Potential applications include a novel 4-port distributed microstrip circulator which may have advantages over junction devices at millimetric wavelengths. Optimum normal-mode conditions for the use of the FCL as a component in the distributed circulator are derived, and the design procedure for microstrip FCL is presented for the first time.**

I. Introduction

At MILLIMETRIC wavelengths, the "drop-in" technology used to fabricate the classical junction circulator in microstrip becomes very demanding and expensive due to the dependence of the circulator's diameter on the wavelength. Moreover, the limitation in available saturation magnetization values and the strong bias field required to overcome the large demagnetization factor of the thin ferrite disk in the direction perpendicular to the plane of the disk present some difficulties. A possible economic solution is to use a completely new type of (4-port) *distributed circulator* [1]–[3] which has the advantage that it does not suffer the size-constraints of the junction circulator and requires only a weak biasing field due to its in-plane magnetization. This device consists of a pair of axially-magnetized Ferrite-Coupled Lines (FCL) in cascade with a 180° hybrid coupler. A possible implementation of this device in planar, microstrip form is proposed in Fig. 1, together with a summary of its operation.

This novel circulator has arisen out of the experimental work of Davis and Sillars [1], and the theoretical work of Mazur and Mrozowski [4], [7]: Early experimental work by Davis and Sillars showed nonreciprocal behavior for a variety of structures employing a pair of lines loaded with axially-magnetized ferrite, including a 4-port device which exhibits circulator behavior [1]. Mazur and Mrozowski (MM) later showed that the operation of the device could be explained in terms of the cascade of a reciprocal 4-port component such as a 180° hybrid coupler with a section of ferrite-coupled lines (FCL) [4]. MM also used the coupled-mode method of Marcuse [5] and Awai and Itoh [6] to show that the behavior of the FCL section was due to coupling between the even- and odd-modes of the pair of symmetrical lines when the ferrite was magnetized longitudinally [7]. Subsequent experimental results [2], [3] confirmed this coupled-mode understanding.

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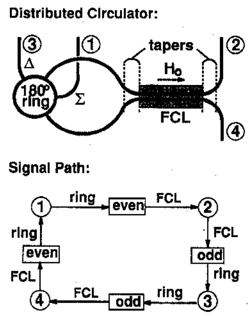

Fig. 1. Schematic diagram of the proposed 4-port distributed microstrip circulator and its signal behavior. The direction of circulation, which has been chosen arbitrarily for the schematic, can be reversed by reversing H_0.

However, satisfactory optimization of this novel circulator has yet to be achieved. This is because:

1) As yet there is no analysis of the impedance(s) of the FCL, without which the matching tapers in Fig. 1 cannot be designed
2) The coupled-mode method is accurate only for weak coupling [8], i.e., for weakly-magnetized ferrite. It will be desirable to shorten the length of the FCL for which stronger coupling is required, and hence a more accurate method of analysis will be needed
3) Suitable geometry and positioning of the ferrite material for different waveguiding structures are not yet well understood.

Hence, the purpose of this study is to present, for the first time [9], an alternative and complementary viewpoint on the behavior of the FCL: as the superposition of two circularly- or elliptically-polarized normal modes of the *magnetized* structure. This normal-mode approach can be used to solve the above difficulties, as indicated below:

1) It is the impedances of the two normal modes that are required. Recent developments in the finite element method [10] enable the E- and H-field vectors of structures containing axially-magnetized ferrite to be calculated, from which the normal-mode impedances can be obtained
2) This approach should give better accuracy as no assumptions about coupling strength are made. The finite element method of [10] also enables the propagation characteristics of the FCL to be calculated

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3) Lastly, effectiveness and positioning of the ferrite material can be estimated from the circularly- or elliptically-polarized field plots of the normal modes, and also from their power-density plots. The computed field solutions of initial structures can give an insight into suitable placement of the ferrite material so that modifications can be made to these structures for further computation. Results of these computations will give further insight which can be used for further modification, and so on, in an iterative design loop.

The normal-mode approach, therefore, *must* be employed in order to achieve satisfactory optimization of the FCL and its matching tapers so that the distributed circulator, implemented either as in Fig. 1 or using other structures, might be competitive with the junction circulator at the higher microwave frequencies.

II. Theory

This section relates the normal-mode viewpoint to the coupled-mode viewpoint of the FCL, and introduces optimal normal-mode conditions that are complementary to the coupled-mode conditions.

A. Optimum Coupled-Mode Conditions

MM have shown that the even- and odd-modes of the *unmagnetized* FCL or "basis guide" become coupled when longitudinal magnetization is applied to the ferrite [7]. By assuming weakly-magnetized ferrite ($\mu_{x,y} \approx 1$ in the permeability tensor), conditions for correct operation of the magnetized FCL are [7]: equal propagation constants for the "basis guide"

$$\beta_{\text{even}} = \beta_{\text{odd}} \tag{1}$$

for which the required FCL length L is given by

$$CL = \frac{\pi}{4} + \frac{n\pi}{2}, \qquad n = 0, 1, 2, \ldots. \tag{2}$$

where the coupling coefficient C caused by the gyrotropy of the ferrite can be calculated from

$$C = \frac{1}{2} k_0 \eta_0 \kappa \int_{\Omega_0} (H_x^{e*} H_y^o - H_y^{e*} H_x^o)\, d\Omega_0 \tag{3}$$

for which Ω_0 is the cross-section of the ferrite material, $H^{e,o}$ are the even- and odd-mode magnetic field vectors of the unmagnetized FCL, k_0 and η_0 are the wavenumber and intrinsic impedance of free space, respectively, and κ is the off-diagonal element of the permeability tensor. Due to the κ term, the sign of C will depend on the parallel or antiparallel direction of magnetization, and this will in turn determine the direction of circulation of the 4-port circulator containing the FCL. Note that, for consistency with the rest of this paper, a few symbols have been changed from the ones used in [7], without any change to the meaning of the equations.

B. From Coupled Modes to Normal Modes

Consider first a pair of symmetrical isotropic lines which are brought close together such that when a signal is fed into one line, there is a periodic transfer of signal power to-and-fro from one line to the other as it propagates down the lines. This periodic transfer of power can be explained either as the coupling which occurs between the individual lines when they are brought together, using the coupled-mode theory [8], or as the superposition of the (normal) even- and odd-modes which are supported by the structure due to the presence of two parallel lines. Hence there exists two consistent and complementary viewpoints on the same phenomenon. Similarly, for the FCL section there should exist two normal modes of the *magnetized* structure resulting from the gyrotropic coupling between the even- and odd-modes of the "basis guide," whose superposition defines the behavior of the FCL.

The normal modes of the symmetrical FCL can be extracted from its coupled-mode equations in [7] by taking advantage of the *orthogonality* property of the normal modes, which states that the average power of each normal mode is distinct and decoupled from that of other normal modes of the same structure, and is therefore *constant*. Hence by forcing the average power $P_{\text{line}}(z)$ on each line of the FCL to be constant, i.e.,

$$\frac{\partial P_{\text{line}}(z)}{\partial z} = 0 \tag{4}$$

solutions for the voltages on each line would then represent the normal modes of the structure.

Let $\angle V_{\text{line,mode}}(z)$ represent the phase of the voltage on each line for each of the two normal modes of the magnetized FCL. Continuing from the coupled-mode equations for weakly-magnetized symmetrical FCL in [7], it can be shown that the phase difference ϕ_{mode} between lines 1 and 2 for the normal modes are

$$\phi_1 = \angle V_{11}(z) - \angle V_{21}(z) = -2 \arctan\left(\frac{\Gamma - \Delta\beta}{C}\right) \tag{5}$$

$$\phi_2 = \angle V_{12}(z) - \angle V_{22}(z) = +2 \arctan\left(\frac{\Gamma + \Delta\beta}{C}\right) \tag{6}$$

where

$$\Gamma = \sqrt{\Delta\beta^2 + |C|^2} \tag{7}$$

$$\Delta\beta = \frac{\beta_{\text{even}} - \beta_{\text{odd}}}{2} \tag{8}$$

and C is the coupling coefficient caused by the gyrotropy of the ferrite, given in (3). Subtracting (5) from (6), it can be shown that

$$|\phi_2 - \phi_1| = 180^\circ \tag{9}$$

independent of $\Delta\beta$ and C. However, (5) and (6) show that individually, ϕ_1 and ϕ_2 *do* depend on $\Delta\beta$ and C. It is interesting that the phase difference between lines for each mode of the magnetized FCL is frequency-dependent, unlike isotropic lines with dispersionless materials which *always* have a 0° (even) and 180° (odd) phase difference.

TABLE I
Expressions for the Voltages and Currents of Each Line and Mode

	mode 1	mode 2
line 1	$V_{11}(z) = V_{p11}e^{-j\beta_1 z}$	$V_{12}(z) = V_{p12}e^{-j(\beta_2 z+\theta)}$
	$I_{11}(z) = \frac{V_{p11}}{Z_{11}}e^{-j\beta_1 z}$	$I_{12}(z) = \frac{V_{p12}}{Z_{12}}e^{-j(\beta_2 z+\theta)}$
line 2	$V_{21}(z) = V_{p21}e^{-j(\beta_1 z+\phi_1)}$	$V_{22}(z) = V_{p22}e^{-j(\beta_2 z+\theta+\phi_2)}$
	$I_{21}(z) = \frac{V_{p21}}{Z_{21}}e^{-j(\beta_1 z+\phi_1)}$	$I_{22}(z) = \frac{V_{p22}}{Z_{22}}e^{-j(\beta_2 z+\theta+\phi_2)}$

For optimum operation of the magnetized FCL, the normal-mode equivalent to the $\beta_{\text{even}} = \beta_{\text{odd}}$ condition in (1) is given by

$$\phi_1 = -90^\circ, \qquad \phi_2 = +90^\circ \tag{10}$$

by substituting (1) into (5)–(8). These phase relations can also be derived by starting from a normal-mode viewpoint.

C. Normal-Mode Analysis of General Coupled Lines

We begin by considering the most general case of a pair of parallel lines in an inhomogeneous and anisotropic structure which supports, in the frequency range of interest, two dominant modes with propagation constants β_1 and β_2 and phase difference between lines ϕ_1 and ϕ_2 for modes 1 and 2, respectively. Let us define an arbitrary phase difference θ between modes 1 and 2. With $e^{j\omega t}$ understood, we can define for each line and mode a voltage $V_{\text{line,mode}}(z)$ with peak voltage $V_{p,\text{line,mode}}$, current $I_{\text{line,mode}}(z)$ and impedance $Z_{\text{line,mode}}$. Expressions for the voltages and currents of each line and mode are given in Table I.

Applying superposition of the voltages and currents for modes 1 and 2, the total average power $P_{\text{line}}(z)$ on each line given by

$$P_{\text{line}}(z) = \frac{1}{2}\text{Re}[V_{\text{line}}(z)I^*_{\text{line}}(z)] \tag{11}$$

can be shown to be

$$P_1(z) = \frac{1}{2}\left(\frac{V_{p11}^2}{Z_{11}} + \frac{V_{p12}^2}{Z_{12}}\right) + \frac{1}{2}V_{p11}V_{p12} \times \left(\frac{1}{Z_{11}} + \frac{1}{Z_{12}}\right)\cos(\beta_2 z - \beta_1 z + \theta) \tag{12}$$

$$P_2(z) = \frac{1}{2}\left(\frac{V_{p21}^2}{Z_{21}} + \frac{V_{p22}^2}{Z_{22}}\right) + \frac{1}{2}V_{p21}V_{p22} \times \left(\frac{1}{Z_{21}} + \frac{1}{Z_{22}}\right)\cos(\beta_2 z - \beta_1 z + \theta + \phi_2 - \phi_1). \tag{13}$$

For conservation of power of bounded modes in lossless lines, $[P_1(z) + P_2(z)]$ must be constant. This imposes, simultaneously, **BOTH** of the following conditions

$$V_{p11}V_{p12}\left(\frac{1}{Z_{11}} + \frac{1}{Z_{12}}\right) = V_{p21}V_{p22}\left(\frac{1}{Z_{21}} + \frac{1}{Z_{22}}\right) \tag{14}$$

$$|\phi_2 - \phi_1| = 180^\circ. \tag{15}$$

The above $|\phi_2 - \phi_1| = 180^\circ$ equation (also from (9)) is therefore a general condition which must hold in all cases where the two modes are bounded and lossless. All these equations are applied to the more specific case of the symmetrical FCL in the next part.

D. Normal-Mode Analysis of Symmetrical FCL

If the lines are symmetrical as in the FCL, for each mode we can make $V_{p11} = V_{p21} = V_{pm1}$, $V_{p12} = V_{p22} = V_{pm2}$, $Z_{11} = Z_{21} = Z_{m1}$, and $Z_{12} = Z_{22} = Z_{m2}$, where "$m1$" stands for "mode 1" and "$m2$" stands for "mode 2". Let us introduce a quantity k to represent the ratio of peak-voltages, given by $k = V_{pm2}/V_{pm1}$. Substituting these quantities into (12) and (13), the average powers simplify to

$$P_1(z) = \frac{1}{2}V_{pm1}^2\left(\frac{1}{Z_{m1}} + \frac{k^2}{Z_{m2}}\right) + \frac{1}{2}V_{pm1}^2 k \times \left(\frac{1}{Z_{m1}} + \frac{1}{Z_{m2}}\right)\cos(\beta_2 z - \beta_1 z + \theta) \tag{16}$$

$$P_2(z) = \frac{1}{2}V_{pm1}^2\left(\frac{1}{Z_{m1}} + \frac{k^2}{Z_{m2}}\right) - \frac{1}{2}V_{pm1}^2 k \times \left(\frac{1}{Z_{m1}} + \frac{1}{Z_{m2}}\right)\cos(\beta_2 z - \beta_1 z + \theta). \tag{17}$$

Conditions that must be imposed for correct operation of the FCL section of length L are: (i) all the power is fed into line 1 at $z = 0$, i.e., $P_2(0) = 0$ and (ii) the voltages at $z = L$ are either even or odd, i.e., $V_1(L) = \pm V_2(L)$, with equal power division $P_1(L) = P_2(L)$ between the lines. Implicit in condition (i) is the condition $\frac{\partial P_2(0)}{\partial z} = 0$ as well, which when applied to (17) gives $\theta = 0$.

Applying $P_2(0) = 0$ to (17) with $\theta = 0$, it can be shown that there exist two solutions for k

$$k = 1, \frac{Z_{m2}}{Z_{m1}} \tag{18}$$

which can be understood to be either the $V_{pm1} = V_{pm2}$ or the $I_{pm1} = I_{pm2}$ case. Let us use the former case for condition (ii). When $P_1(L) = P_2(L)$, we obtain the normal-mode expression for L which is equivalent to (2)

$$(\beta_1 - \beta_2)L = \frac{\pi}{2} + n\pi \text{ rad}, \qquad n = 0, 1, 2, \ldots \tag{19}$$

With these values, the total voltage $V_{\text{line}}(L)$ at the end of each line of the FCL can be written from Table I as

$$V_1(L) = V_{pm1}e^{-j\beta_1 L}[1 + e^{-j(\beta_2 - \beta_1)L}] \tag{20}$$

$$V_2(L) = V_{pm1}e^{-j(\beta_1 L + \phi_1)}[1 - e^{-j(\beta_2 - \beta_1)L}] \tag{21}$$

from which it can be shown that for the n integer in (19), the phase difference between lines at $z = L$ depends only on ϕ_1 (which also defines ϕ_2 according to (15))

$$\angle V_1(L) - \angle V_2(L) = \begin{cases} \phi_1 - \pi/2, & \text{for } n = 0, 2, 4, \ldots \\ \phi_1 + \pi/2, & \text{for } n = 1, 3, 5, \ldots \end{cases}. \tag{22}$$

If we have, say, the "odd-mode," $V_1(L) = -V_2(L)$ at the output of the FCL, then the values of ϕ_1 and ϕ_2 will be

$$\phi_1 = -90^\circ, \qquad \phi_2 = +90^\circ \tag{23}$$

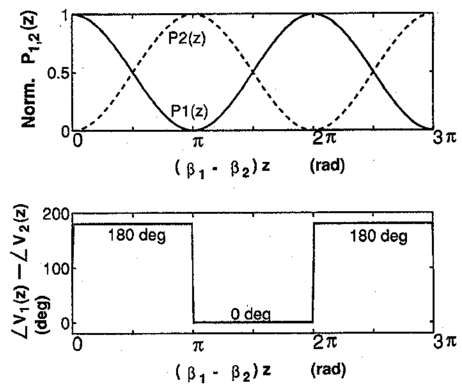


Fig. 2. Normalized powers $P_1(z)$, $P_2(z)$ and phase difference $[\angle V_1(z) - \angle V_2(z)]$ of symmetrical FCL when $\phi_1 = -90°$, $\phi_2 = +90°$.

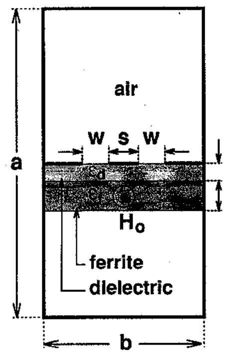


Fig. 3. Cross section of FCL using coupled slotlines. Dimensions in millimeters are: $a = 7.2$, $b = 3.4$, $w = s = 0.5$, $h_1 = 0.127$, $h_f = 0.5$. The permittivities are: finline substrate $\epsilon_d = 2.22$, ferrite slab $\epsilon_f = 13.5$, and saturation magnetization $M_s = 340$ kA/m. Ferrite is just saturated ($H_0 = 0$ Oe) (data follows that of Mazur [11]).

which, as in (10), is the normal-mode equivalent to the coupled-mode condition $\beta_{\text{even}} = \beta_{\text{odd}}$. The expressions in (22) can also be applied to symmetrical isotropic lines, for which $\angle V_1(L) - \angle V_2(L) = \pm 90°$.

The graphs of $P_1(z)$, $P_2(z)$ and $[\angle V_1(z) - \angle V_2(z)]$ under these conditions are shown in Fig. 2, where it is seen that the phase difference between lines 1 and 2 is always either 0° or 180°, so that when they have equal power either the "even-" or "odd-mode" is seen. The effect of reversing the direction of magnetization of the FCL must be obtained from a numerical field solution, but the same kind of nonreciprocal effects as in [7] should be observed.

III. Coupled Slotlines on Ferrite

The coupled slotline structure in Fig. 3 was analyzed by Mazur for its "basis guide" even- and odd-modes [11], from which the magnetized FCL section of a 3-port distributed circulator was designed using the coupled-mode conditions in (1)–(3). We have solved for both the magnetized (gyrotropic) and unmagnetized (isotropic) normal modes of the structure, using an E-field formulation of the finite element method (FEM) in [10] for the magnetized case. Our numerical results are shown in Figs. 4–7. The dispersion diagram for the coupled-slots in Fig. 4 indicates that coupling is present. However, the RHCP-mode curve is closer to the even- and

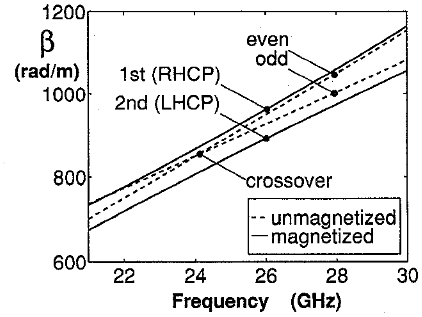


Fig. 4. Coupled slotlines—Dispersion diagram of the FCL with and without magnetization, where β is the propagation constant. The ferrite is just saturated for the magnetized case ($H_0 = 0$ Oe).

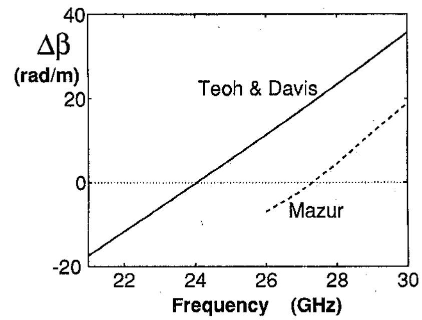


Fig. 5. Coupled slotlines—Variation of $\Delta\beta = (\beta_{\text{even}} - \beta_{\text{odd}})/2$ of the unmagnetized structure with frequency. Zero crossovers are at 24 GHz (Teoh and Davis) and 27.3 GHz (Mazur).

odd-mode curves compared to the LHCP-mode, contrary to the symmetrical curves that would be obtained using the coupled-mode theory due to the *form* of the equations for β_{RHCP} and β_{LHCP} (from [7] with some change in symbols)

$$\beta_{\text{RHCP}}, \beta_{\text{LHCP}} = \frac{\beta_{\text{even}} + \beta_{\text{odd}}}{2} \pm \sqrt{\Delta\beta^2 + |C|^2} \qquad (24)$$

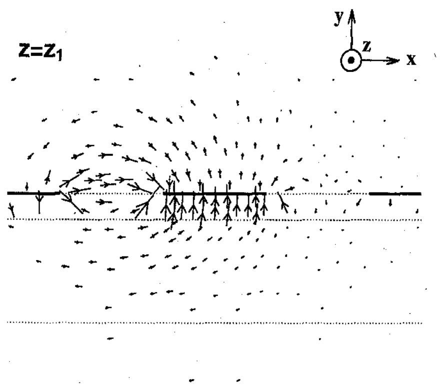
where $\Delta\beta$ and C are defined in (8) and (3), respectively.

In Fig. 5, a comparison is made between our values of $\Delta\beta$ with Mazur's for the unmagnetized structure, and it can be seen that our curve is higher by ≈20 rad/m, giving $\Delta\beta = 0$ at 24 GHz compared with Mazur's value of 27.3 GHz. This may be attributed to the high sensitivity of the crossover frequency to errors in β_{even} and β_{odd}.

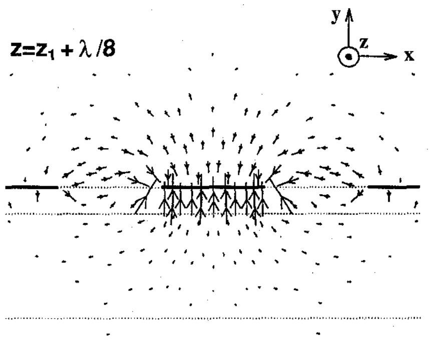
Fig. 6(a)–(c) show the transverse E-field of the first, RHCP mode of the magnetized structure at 23 GHz, at successive values of z which are $\lambda/8$ apart. The minima of the slots are $\approx \lambda/4$ apart, hence giving $\phi_1 \approx -90°$. Values of $\phi_{1,2}$, obtained from observation of the field vectors in this fashion, are plotted in Fig. 7, with the normal-mode condition $\phi_{1,2} = \pm 90°$ occurring at ≈23 GHz, close to the value of 24 GHz when $\Delta\beta = 0$. Mazur's values of $\phi_{1,2}$ were calculated indirectly by substituting his values of $\Delta\beta$ and C from Fig. 4 in [11] into (5) and (6). Note that our values of $\phi_{1,2}$, obtained independently from the fields of each mode, confirm (15).

IV. Coupled Microstrip on Ferrite

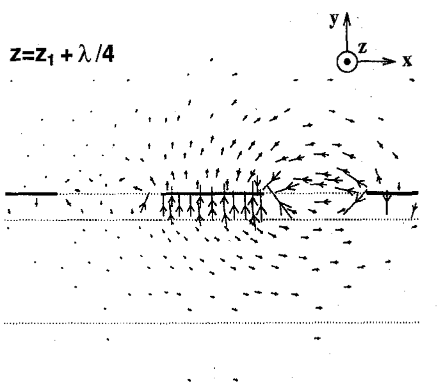
So far there has not been any discussion on the implementation of the distributed 4-port circulator in microstrip, which



(a)



(b)



(c)

Fig. 6. Coupled slotlines—Transverse E-field of mode 1 of the magnetized structure at 23 GHz, (a) at a plane $z = z_1$, (b) at $z = z_1 + \lambda/8$, and (c) at $z = z_1 + \lambda/4$.

must be considered in view of its widespread use and the possibility of compatibility with MMIC's. Questions that need to be answered are: i) How can the condition $\beta_{\text{even}} = \beta_{\text{odd}}$ be achieved in microstrip? ii) Is the resulting microstrip FCL of a reasonable length?

A. Design Procedure for the FCL

The procedure for the design of the FCL section is:

1) Use the FEM to obtain the dispersion diagram of the even- and odd-modes, to determine the crossover frequency of the two modes

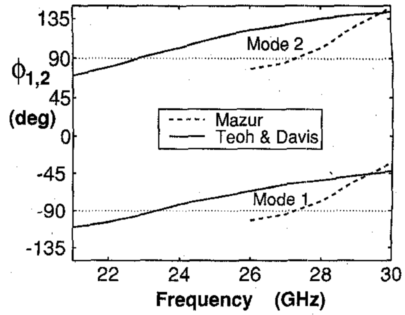


Fig. 7. Coupled slotlines—Variation with frequency of the phase difference between lines, ϕ_{mode}, for each normal mode of the magnetized structure.

2) At this frequency, use the FEM to solve for the field distributions (eigenvectors) of the circularly-polarized normal-modes when magnetization is applied, to check that the $\pm 90°$ condition is fulfilled. Their corresponding propagation constants (eigenvalues) will then give us the required length L of the FCL at this frequency using (19).

For the first step of determining the optimum frequency, it is easier to try to achieve $\beta_{\text{even}} = \beta_{\text{odd}}$ instead of $\phi_{1,2} = \pm 90°$, as the frequency-dependence of β_{even}/k_0 and β_{odd}/k_0 can be understood from the point of view of the frequency-dependence of their *effective permittivities*, whereas the frequency-dependence of $\phi_{1,2}$ is not yet well understood. For the second step of determining the length L, the normal-mode analysis should be used to obtain L because of its greater accuracy especially for strong coupling, when the coupled-mode method is inaccurate because of its *weak coupling* assumption.

As we are considering the propagation characteristics of microstrip lines on ferrite, an important quantity is the effective relative permeability, μ_{eff}, of the ferrite when magnetized longitudinally. It varies with frequency according to [12]

$$\mu_{\text{eff}} = \frac{\mu^2 - \kappa^2}{\mu} \tag{25}$$

where μ and κ are elements in the relative permeability tensor, and are defined in [13]. Fig. 8 shows that μ_{eff} falls as frequency is reduced and as $|\kappa/\mu|$ increases. The coupled-mode method of MM assumes weak coupling of the even- and odd-modes, so that it has the inherent assumption that $\mu_{\text{eff}} \approx 1$, i.e., operation of the FCL is at frequencies far above resonance. When considering propagation characteristics for larger values of $|\kappa/\mu|$ nearer resonance, this assumption is no longer valid, and the coupled-mode method in [7] needs to be modified accordingly. The above design procedure for the FCL is still applicable, but using these modified coupled-mode equations. The handling of the effect of μ_{eff} on the propagation characteristics of coupled microstrip lines on ferrite is best shown through the following design.

B. Numerical Results

The cross section of the coupled microstrip structure under consideration is shown in Fig. 9. The decision to place the ferrite below the strips rather than above them is because it has been found that coupling of the even- and odd-modes is

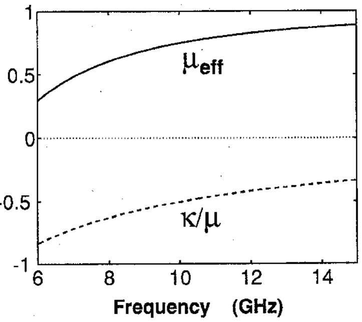


Fig. 8. Coupled microstrips—Variation of effective permeability (μ_{eff}) and gyrotropy ratio (κ/μ) with frequency for $4\pi M_s = 1800$ Gauss and $H_0 = 0$ Oe. The value of μ_{eff} reduces the value of β/k_0 at lower frequencies.

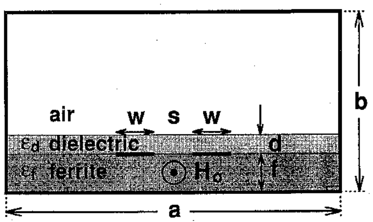


Fig. 9. Cross section of coupled microstrip lines on an axially-magnetized ferrite substrate with dielectric overlay. Dimensions in millimeters are: $w = s = f = 1.0$, $d = 0.5$, $a = 11$ and $b = a/2$. Overlay has a dielectric constant $\epsilon_d = 10$. Ferrite substrate is Trans-Tech TT 2-4097 Nickel ferrite with saturation magnetization $4\pi M_s = 1800$ Gauss and dielectric constant $\epsilon_f = 10$.

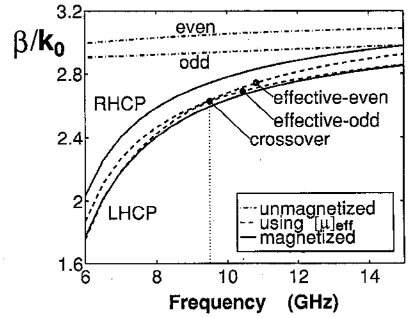


Fig. 10. Coupled microstrips—Normalized propagation constant β/k_0 versus frequency for three cases: 1) unmagnetized (isotropic), 2) using $[\mu]_{\text{eff}}$ which has zero off-diagonals and diagonal elements $\mu_x = \mu_y = \mu_{\text{eff}}$ (anisotropic), and 3) magnetized, with $4\pi M_s = 1800$ Gauss and $H_0 = 0$ Oe (gyrotropic, hence RHCP and LHCP). The crossover of the effective-even and effective-odd modes occurs at ≈ 9.5 GHz.

weak when the ferrite is above the strips. The stronger coupling in the former case is expected as the fields are concentrated in the region beneath the strips.

The normalized propagation characteristics of the structure are shown in Fig. 10, where β is the propagation constant and $k_0 = 2\pi f\sqrt{\mu_0\epsilon_0}$ for frequency f. The normalized dispersion diagram was obtained by using the finite element method of [10], solving for the E-field, with 214 lowest-order edge elements, and 126 axial and 341 tangential nodes. After applying constraints at the E-walls, the S and T matrices of the eigenvalue problem were of order 387-by-387. The propagation characteristics are shown for three cases: 1) with the ferrite unmagnetized and isotropic, 2) with μ_{eff} used in the modified relative permeability tensor $[\mu]_{\text{eff}}$, for $4\pi M_s = 1800$ Gauss and $H_0 = 0$ Oe, so that $[\mu]_{\text{eff}}$ is anisotropic but not gyrotropic, and 3) using the gyrotropic tensor $[\mu]_{\text{gyro}}$ for longitudinal magnetization with $H_0 = 0$ Oe. In case 2), the relative permeability tensor is changed from $[\mu]_{\text{gyro}} \Rightarrow [\mu]_{\text{eff}}$ as in (26)

$$\begin{bmatrix} \mu & +j\kappa & 0 \\ -j\kappa & \mu & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} \mu_{\text{eff}} & 0 & 0 \\ 0 & \mu_{\text{eff}} & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad (26)$$

Fig. 10 shows that as frequency reduces, the difference between the propagation constants of the RHCP and LHCP modes of the magnetized structure and the even- and odd-modes of the unmagnetized structure becomes very large due to the fall in the value of μ_{eff} at lower frequencies (Fig. 8). The coupled-mode method of MM [7] needs to be modified to accommodate this effect if it is to be used at frequencies nearer to cut-off, especially for microstrip. Although the use of μ_{eff} gives good results for single ferrite lines, for ferrite-coupled lines the absence of gyrotropy in $[\mu]_{\text{eff}}$ forces the two dominant modes to remain even and odd so that coupling does not occur. The LHCP and RHCP modes when $[\mu]_{\text{gyro}}$ is used will then be perturbations of the modes when $[\mu]_{\text{eff}}$ is used, as shown clearly in the dispersion diagram. Hence it is to these two *effective-even* and *effective-odd* modes when $[\mu]_{\text{eff}}$ is used that the coupled-mode method should be applied, and the coupled-mode condition we are seeking can be rewritten as

$$\beta_{\text{even}}^{\text{eff}} = \beta_{\text{odd}}^{\text{eff}}. \quad (27)$$

Equalization of the effective-β's has been achieved firstly by adding a dielectric overlay which has the same dielectric constant as the ferrite (see Fig. 9), thereby bringing the even- and odd-modes of the unmagnetized structure close together, but with the even-mode always having a slightly higher β/k_0 due to its having more field concentration in the ferrite region. However, this also means that for the dispersion curves of the effective-even and effective-odd modes, the fall in μ_{eff} as frequency decreases has a greater effect on the effective-even mode, so that at some frequency the effective-even and effective-odd modes must cross. In Fig. 10, this occurs at ≈ 9.5 GHz.

The phase difference $\phi_{1,2}$ between lines for each mode is plotted in Fig. 11, where it is shown that the optimum normal-mode condition $\phi_{1,2} = \pm 90°$ occurs at ≈ 9.8 GHz, close to the previous value of 9.5 GHz and justifying the use of the modified coupled-mode condition of (27). The value of $|\phi_2 - \phi_1|$ remains always $\approx 180°$, confirming (15) again. The values of $\phi_{1,2}$ were obtained by comparing the phase of the vertical-component of the E-field vectors in the region *immediately* beneath each strip, which are the field vectors most closely associated with each strip. Inspection of Fig. 12(a)–(c) at a mid-point in the ferrite layer shows that mode 1 is an RHCP mode, and that at 9.8 GHz the strips have a phase difference of $\approx 90°$. These field vector distributions help to show that the phase difference between the lines *supports* the circular-polarization of the transverse-field vectors in the region of the ferrite between the strips.

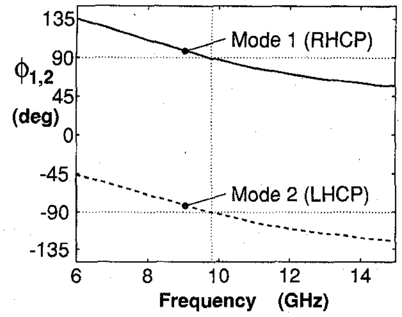


Fig. 11. Coupled microstrips—Phase difference between lines, ϕ_1 and ϕ_2, for modes 1 (RHCP) and 2 (LHCP), respectively, when $H_0 = 0$ Oe. The RHCP and LHCP modes have $\phi_{1,2} = \pm 90°$ at $\approx$9.8 GHz.

The next matter of concern is the required length of the microstrip FCL. At 9.8 GHz, $[\beta_{\text{RHCP}} - \beta_{\text{LHCP}}] = 30.05$ rad/m from the dispersion diagram. Applying (19) with $n = 0$, this gives a FCL length $L = 52.3$ mm. The value of $[\beta_{\text{RHCP}} - \beta_{\text{LHCP}}]$ remains within 29.8–31.5 rad/m over the frequency range 7.5–11 GHz, but the bandwidth of the FCL is limited by $\phi_{1,2}$, which varies at a rate of $\approx 11°$/GHz, from Fig. 11.

Last, the structure of Fig. 9 may be used at higher frequencies by increasing H_0, which will shift the dispersion curves of the effective-even, effective-odd, RHCP and LHCP modes up to higher frequencies.

V. Conclusion

In this paper we have shown, for the first time [9], that in addition to the explanation of the behavior of the magnetized FCL as the gyrotropic coupling between the even- and odd-modes of its "basis guide" [7], there also exists an alternative and complementary normal-mode viewpoint which describes the behavior of the FCL as the superposition of two circularly- or elliptically-polarized normal modes of the magnetized structure. This normal-mode method of analyzing structures containing ferrite has become possible due to recent developments in the finite element method [10], and has several advantages over the coupled-mode method, such as the potential to calculate the impedance(s) of the FCL and the optimum placement of ferrite material.

We have shown that the normal modes can be derived from the coupled-mode equations in [7] through the use of the orthogonality property of normal modes to give expressions for the phase difference $\phi_{1,2}$ between lines for each normal mode. These expressions show that $\phi_{1,2}$ is frequency-dependent for the magnetized FCL structure, whereas for isotropic lines it would always remain constant at 0°/180°. Observation of the E-field distributions of these normal modes for microstrip FCL shows that this unusual phase lead/lag of the lines is necessary in order to *support* the circular/elliptical polarization of the transverse-field vectors in the region between the lines, which themselves are a result of the interaction between the field and the ferrite.

A theoretical normal-mode analysis of a general pair of coupled lines has produced the general condition $|\phi_2 - \phi_1| = 180°$ which is always true for a pair of bounded and lossless modes. This normal-mode analysis has also been applied to

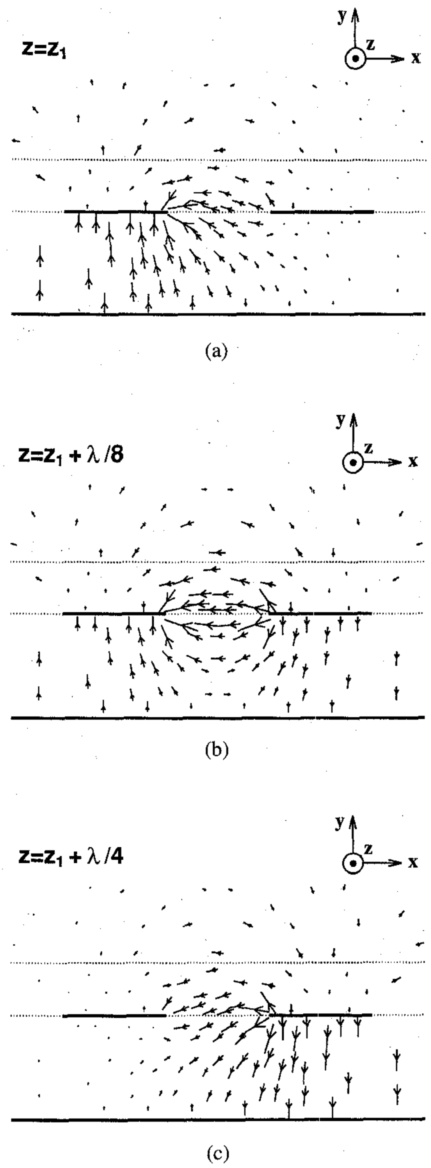


Fig. 12. (a) Coupled microstrips—Transverse E-field of mode 1 of the magnetized structure at 9.8 GHz, (a) at a plane $z = z_1$, (b) at $z = z_1 + \lambda/8$, and (c) at $z = z_1 + \lambda/4$.

symmetrical FCL, and the following normal-mode conditions for optimum operation of symmetrical FCL of length L, from (19) and (23), have been obtained

$$(\beta_1 - \beta_2)L = \frac{\pi}{2} + n\pi \text{ rad}, \qquad n = 0, 1, 2, \ldots$$
$$\phi_{1,2} = \pm 90°.$$

The $\phi_{1,2} = \pm 90°$ condition is presented for the first time [9].

A general procedure has been outlined for the design of the FCL, which takes advantage of the availability of both coupled-mode and normal-mode approaches: the first step is to achieve $\beta_{\text{even}} = \beta_{\text{odd}}$ at the frequency of interest, followed by a normal-mode calculation of the dispersion diagram to obtain the FCL length L and a normal-mode field vector solution at the design frequency to check that $\phi_{1,2} = \pm 90°$ is fulfilled at that frequency.

Also, for the first time, it has been shown that it is possible to implement the FCL in microstrip, using the above procedure for the design, but with a modification in the coupled-mode condition that is sought. This is because the effective relative permeability μ_{eff} of longitudinally-magnetized ferrite used in microstrip, given by $\mu_{\text{eff}} = (\mu^2 - \kappa^2)/\mu$, has been shown to have a large influence on the dispersion diagram of the microstrip FCL, such that the concept of *effective-even* and *effective-odd* modes has to be introduced in order to aid the design of the microstrip FCL. These modes can be solved for using a modified relative permeability tensor $[\mu]_{\text{eff}}$ for which $\mu_x = \mu_y = \mu_{\text{eff}}$, $\mu_z = 1$, and the off-diagonal elements are made zero. This modified tensor is not gyrotropic, therefore forcing the modes to remain even and odd so that coupling does not occur, although the general trend of the dispersion curves should follow quite closely to that of the magnetized FCL which is solved using the actual gyrotropic tensor $[\mu]_{\text{gyro}}$, especially when far away from regions of strong coupling. The use of $[\mu]_{\text{eff}}$ effects a kind of "de-coupling" of the magnetized structure. Hence, the optimal normal-mode conditions are fulfilled at the frequency at which the effective-even and effective-odd modes cross over, so the modified coupled-mode condition that should be sought initially in the design is, from (27), $\beta_{\text{even}}^{\text{eff}} = \beta_{\text{odd}}^{\text{eff}}$. The dispersion diagrams of the microstrip FCL verify the validity of this proposal.

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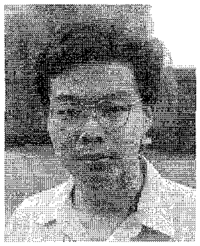

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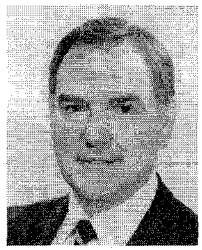

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